Calvin: "Here's another math problem I can't figure out. What's 9 + 4?"
Hobbes: "Ooh, that's a tricky one. You have to use calculus and imaginary numbers for this."
Calvin: "IMAGINARY NUMBERS?!"
Hobbes: "You know, eleventeen, thirty-twelve, and all those. It's a little confusing at first."
Calvin: "How did YOU learn all this? You've never even gone to school!"
Hobbes: "Instinct. Tigers are born with it."

-- Bill Watterson, Calvin and Hobbes

Thankfully, we don't have to be mathematical geniuses to successfully keep honey bees. However there are fundamental relationships in the hive, ones that we need to understand, that can be expressed as simple formulas.

Worker brood: 3/6/12

A developing worker bee spends about **3 days** as an egg, **six days** as a larva and **twelve days** as a pupa.

How is this information useful? Assume we inspect our hive and we see two frames of capped brood. How many larvae should we expect? At steady state, we'll have half as much larvae (open brood) as pupae (capped brood). And we'll have half as many eggs as larvae.

What if we have far more capped brood than twice the amount of larvae? That suggests that brood-rearing is slowing down. If we see this in the fall, that is normal and expected. If it is the spring, something is wrong… is the queen present? Is the hive preparing to swarm? Is there lots of pollen stored in the cells (critical for brood-rearing), or is it all gone?

Swarm math

A developing queen bee spends about 3 days as an egg, just like worker bees. She is capped about a half day earlier than workers, after about 5-1/2 days as a larva, and rapidly matures with only about 6-1/2 days as a pupa. The adult virgin emerges, on average, after about 16 days.

This 10 pound swarm settled in one of my plum trees. Roughly speaking, about how many bees is that?

She spends three or so days wandering around the hive, strengthening her wings and doing further maturing, before going on her first mating flight. She will go on several mating flights over several days, weather permitting. Then she requires a couple of days to begin laying. So it can easily take two weeks between the emergence of a new queen and the first egg.

If the new queen is a swarm queen, half the colony and the old mother queen will have left the hive sometime after the first queen cell was capped. Even before that, the colony will have put the old mother queen on a fitness plan, reducing her weight so she can fly with the swarm. This will also cause her to stop laying eggs. Therefore the last eggs in the colony will have been laid sometime in the week before the first queen cell was capped.

So the last worker brood will emerge before the new queen begins laying eggs. A beekeeper who inspects the hive during this interval will find a reduced population, no sealed queen cells, no eggs, no larvae, no capped worker brood and just maybe a few capped drone brood cells. The common diagnosis? The colony is queenless! Maybe the drone brood is from laying workers! *AAAAAARG!!!*

Wait… everything is actually progressing well, precisely according to plan, but there isn't glaring evidence of it. The period when a hive appears queenless but is actually on the verge of having a new, vigorously laying queen is what

I call the Twilight Zone. Is the colony coming or going? Should I do nothing or should I scramble around like a maniac? This question can trip up beginners and old-timers alike. This situation was discussed in depth earlier (see I need to buy a queen! Or do I?).

The practical math here is to wait a month after a colony has swarmed before expecting to see any tangible evidence of the new queen. If we don't see any eggs or larvae after a month, wait a week longer to make sure. However as a bit of insurance, at any time we can take a frame with eggs and larvae from another hive and give it the maybe-queenless hive. If actually queenless, the bees will begin constructing queen cells on the donor comb. If not queenless, we've simply boosted the population a bit, not a bad thing.

Brood-to-bee ratio

One bee inside a cell covers three cells when outside the cell. So if we have one frame completely full of brood, the brood on it will emerge to give us three full frames of bees.

Cells per frame, bees per frame

The worker-sized deep foundation I use is 42 cells tall and 79 cells wide, for a total of 3,318 cells per side. A commonly cited number is 3,500 cells per side; that makes the math easier so let's go with that. With 3,500 cells per side, that gives us 7,000 cells per deep frame.

The commonly cited number of bees in a three-pound package is 10,000 bees, although this is just a convention and not a firm measurement. Regardless of its precision, it is good enough for us to use for comparison purposes.

So, if there are around 10,000 bees in a 3-pound commercial package, that is equivalent to the brood from 3 sides of frames, or 1½ frames (3,500 x 3 = 10,500). Going back to our brood-to-bee ratio, 1½ frames of brood, when emerged, should cover 4½ frames with adult bees (1.5 x 3 = 4.5). Does that sound about right? These are rough rules of thumb and shouldn't be used as the basis for lawsuits, but they seem reasonable to me.

This feral colony was in a house near Graham. Judging by the relative area of comb space and the number of combs covered by bees, can you guesstimate how many bees it contained?

Looking at this from a different angle, if there are 3,500 cells per frame face and one bee covers 3 cells, there can be more than 1,100 bees on a frame face, or 2,200 on a full frame.

This gives us a rough way to evaluate our colonies relative to packages or nucs. If we inspect a hive in the spring and the bees only cover two full frames, they are half the strength of a brand new package or nuc. Not good. If they fully cover nine frames, there are about 20,000 or so of them. Not bad at all for early spring.

Population, birth and death rate

How many eggs does a queen lay in a day? That depends on which queen, which day, the queen's age, the availability of pollen and nectar and so on. So who knows? The often-cited number is around 1,500 eggs per day (on average, one every minute) during the height of the nectar flow.

Given this, how many bees can be in a hive? That depends on the egg-laying rate as well as how long a worker bee lives, which depends on the season of year, the genetics of the bee and many other factors. Often-cited numbers for how long a worker bee lives during spring and summer are four to six weeks; let's go with five.

So if a queen lays 1,500 eggs per day and a worker lives for five weeks (35 days), the colony can easily have more than 50,000 workers

(1,500 x 35=52,500). But remember that this is under peak conditions.

If a queen lays 1,500 eggs per day, 1,500 adult worker bees will emerge three weeks later. Five weeks after that, 1,500 worker bees will die (that's life!). If the weather is bad and the bees can't forage, those bees will die within the hive rather than in transit to or from the hive. As soon as the weather allows, undertaker bees will drag out the dead and dump them in front of the hive. The corpses can be hidden in the grass or carried off by ants, but the observant beekeeper will see many of them.

Frames filled per day

From above, if a queen lays 1,500 eggs per day and one side of a deep frame can potentially contain 3,500 cells, a queen can potentially fill 43% of a deep frame side every day. At that rate it takes nearly five days for a queen to fully fill the equivalent of both sides of a deep frame, and a week to fill a frame and a half. Remember that a frame and a half worth of brood, turned into adult bees, was our rough estimate of the contents of a package or nuc.

Drone to worker ratio

A first-year hive is in Establishment Mode so generally focuses on comb-building, food collection and raising worker brood. After that, the colony shifts to Reproductive Mode and we see swarm cells and lots of drone production.

An established colony will want about 10% of its brood to be drone brood. They see this as an innate biological imperative. If the brood area has nothing but worker foundation, the colony will fit the drone cells wherever it can, such as in the gap between upper and lower frames in stacks of boxes.

One way to help ensure a tidy hive is to insert a frame with special drone-sized foundation into the second or next-to-last position of the brood box. With a ready-made location, drone cells should no longer be forced into inconvenient spaces.

Drone frames can also be used to supplement our varroa mite control strategy. Once the cells are capped, the frame is removed and replaced with a fresh one. The capped frame is frozen to kill the mites that are parasitizing the drone pupae. The frame is then thawed and returned to the hive as part of the next frame rotation. The bees will clean out the dead pupae and the queen will refill the cells with drone eggs.

Sugar math

The sugar concentration of nectar varies considerably from plant species to plant species and also depends on rainfall, temperature etc. The typical concentration is somewhere between 25% and 50% sugar, with sucrose being the primary form of the sugar. Beekeepers typically feed bees 1-to-1 sugar syrup, or a bit thinner, in the spring because that mimics the natural nectar flow. Also, bees dilute honey to about 50% moisture concentration before they can digest it. So a 50% or so solution can be directly used by the bees, which stimulates wax production as well as brood-rearing.

In fall, our goal is for the bees to fill their food pantry as quickly and efficiently as possible. We aren't interested in them being able to immediately use the sugar; we want to assist the bees in drying it to 15% or less moisture for safe long-term storage. So we feed a heavier sugar/water mixture, typically 2-to-1 sugar-to-water. Why not thicker than that? At room temperature, 211.5 grams of sucrose will stay in solution in 100 grams of water, so 2-to-1 is just about as thick as we can get it and have the sugar syrup remain completely liquid. The bees then perform a bit of magic with enzymes to create a super-saturated solution (honey) that is much drier than we can do.

Basic Honey Math

- 16 ounces of honey by weight is 12 ounces by volume
- One pint of honey = 1.5 pounds
- One quart of honey = 3 pounds
- One gallon of honey = 12 pounds
- One 5-gallon bucket of honey = 60 pounds

Even More

These are just a few in-hive relationships and beekeeping factoids that hopefully will warm up the math neurons in your brain. There are many more. Being able to quantify, or at least guesstimate, the implications of what we observe in our hives can only make us better beekeepers. Take notes. Pull out those slide rules and pocket protectors. Embrace your inner nerd! You'll be amazed at what you find.

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